Formulari de Física Estadística

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I. Col·lectivitat Microcanònica

$$S = k_B \log \Omega$$

Maximització:

$$N_s = \sum_{j=1}^{M} n_j \qquad \Omega(n_1, \ldots, n_M) = \frac{N_s!}{\prod_{i=1}^{M} n_i!}$$

$$S = -k_B \sum_i p_i \log p_i \quad p_i = \frac{1}{M} \quad n_j = \frac{N_s}{M}$$

Relacions amb termodinàmica:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V} \qquad C(T) = \frac{\partial E}{\partial T}$$

$$p = T\left(\frac{\partial S}{\partial V}\right)_{E,N} \qquad \mu = -T\left(\frac{\partial S}{\partial N}\right)_{E,V}$$

II. Col·lectivitat Canònica

$$F = -k_B T \log Z$$

Maximització:

$$\sum_{j=1}^{M} n_j = N_s \qquad E_t = \sum_{i=1}^{N_s} E_i = \sum_{j=1}^{M} n_j E_j$$

$$p_j = \frac{e^{-\beta E_j}}{\sum_{j=1}^{M} e^{-\beta E_j}} \quad Z = \sum_{j=1}^{M} e^{-\beta E_j} = \sum_{E_j} g(E_j) e^{-\beta E_j}$$

Valor mig i fluctuacions de l'energia:

$$U = \langle E \rangle = -\left(\frac{\partial \log Z}{\partial \beta}\right)_{N,V} \quad (\Delta E)^2 \equiv \sigma_E^2 = k_B T^2 C_v$$

Relacions amb termodinàmica:

$$S = \frac{U - F}{T} = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_N \qquad \mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

Cas continu:

$$Z_1(N,V,T) = \frac{1}{h^3} \int_{\mathcal{D}} d\vec{p} d\vec{q} e^{-\beta \mathcal{H}(\vec{p}\vec{q})}$$

Funció de partició clàssica:

Independents i indistingibles:

$$Z(N,V,T) = \frac{[Z_1(N,V,T)]^N}{N!}$$

Independents i distingibles:

$$Z(N,V,T) = [Z_1(N,V,T)]^N$$

Teorema d'equipartició:

$$\left\langle x_i \frac{\partial \mathcal{H}}{\partial x_j} \right\rangle = \delta_{ij} k_B T$$

III. Col·lectivitat Grancanònica

$$\Xi = -k_B T \log Z_{GC}$$

Maximització:

$$M' = \sum_{N=1}^{\infty} M(N) \quad \sum_{i=1}^{M'} n_i = N_s \quad \sum_{i=1}^{M'} n_i E_i = E_t$$

$$p_j = \frac{e^{-\beta \varepsilon_j + \beta \mu N_i}}{\sum_{N=1}^{\infty} z^N \sum_{i=1}^{M(N)} e^{-\beta E_i}}$$

$$Z_{GC} = \sum_{N_s=0}^{\infty} \sum_{E_j} z^{N_s} g(E_j) e^{-\beta E_j} = \sum_{N_s=0}^{\infty} Z(N_s, V, T) z^{N_s}$$

$$z = e^{\beta \mu}$$

Valors mitjos i fluctuacions:

$$N \equiv \langle N \rangle = z\left(\frac{\partial \log Z_{GC}}{\partial z}\right)_{T,V} = \frac{1}{\beta}\left(\frac{\partial \log Z_{GC}}{\partial \mu}\right)_{V,\beta}$$

$$(\Delta N)^2 = \sigma_N^2 = k_B T \left(\frac{\partial \langle N \rangle}{\partial \mu}\right)_{T,V}$$

$$U = \langle E \rangle = -\left(\frac{\partial \log Z_{GC}}{\partial \beta}\right)_{Z,V}$$

$$\sigma_E^2 = k_B T^2 \left(\frac{\partial U}{\partial T}\right)_{z,V} = k_B T^2 C_v + k_B T \left(\frac{\partial U}{\partial N}\right)_{T,V} \left(\frac{\partial U}{\partial \mu}\right)_{T,V}$$

Relacions amb termodinàmica:

$$\Xi = U - TS - \mu N$$

$$pV\beta = \log Z_{GC} \quad N = -\left(\frac{\partial \Xi}{\partial \mu}\right)_{T,V} \qquad S = -\left(\frac{\partial \Xi}{\partial T}\right)_{V,\mu}$$

Funció de partició G.C. d'un sistema ideal:

Partícules independents i indistingibles:

$$Z_{GC}(\mu, V, T) = \sum_{N_s=0}^{\infty} \frac{(zZ_1)^{N_s}}{N_s!} = e^{zZ_1}$$

Partícules independents i distinigibles:

$$Z_{GC}(\mu, V, T) = \sum_{N_s=0}^{\infty} (zZ_1)^{N_s} = \frac{1}{1 - zZ_1}$$

IV. Estadístiques Quàntiques

Degeneracions:

$$g_{MB}(n_i) = \frac{N!}{\prod_{i=1}^{M} n_i!} \qquad g_{BE} = g_{FD} = 1$$

Simetria:

$$[P, \mathcal{H}] = 0 \quad P^2 = \pm 1 \quad \lambda = 1 : \mathrm{B} \quad \lambda = -1 : \mathrm{F}$$

Restriccions:
BE:

$$\sum_{i=1}^{M} n_i = N \quad n_i = 0, 1, \ldots, N$$

FD:

$$\sum_{i=1}^{M} n_i = N \quad n_i = 0, 1$$

Funció de partició:

$$Z_N = \sum_{\{n_i\}} \mathrm{e}^{-\beta \sum \varepsilon_i n_i}$$

Funció de partició Grancanònica:

$$Z_{GC} = \sum_{\{n_i\}} \prod_{j=1}^{M} (z\mathrm{e}^{-\beta\varepsilon_j})^{n_j} = \prod_{j=1}^{M} \sum_{\{n_i\}} (z\mathrm{e}^{-\beta\varepsilon_j})^{n_j}$$

BE:

$$Z_{GC} = \prod_{j=1}^{M} \sum_{n_i=0}^{N} (z\mathrm{e}^{-\beta\varepsilon_j})^{n_j} = \prod_{j=1}^{M} \frac{1}{1 - z\mathrm{e}^{-\beta\varepsilon_j}}$$

FD:

$$Z_{GC} = \prod_{j=1}^{M} \sum_{n_i=0}^{1} (z\mathrm{e}^{-\beta\varepsilon_j})^{n_j} = \prod_{j=1}^{M} \left[1 + z\mathrm{e}^{-\beta\varepsilon_j}\right]$$

General:

$$Z_{GC} = \prod_{j=1}^{M} \left(1 + sz\mathrm{e}^{-\beta\varepsilon_j}\right)^{s} \quad s = 1 : \text{FC} \; s = -1 : \text{BE}$$

Relacions amb termodinàmica:

$$pV\beta = s \sum_{j=1}^{M} \log(1 + sz\mathrm{e}^{-\beta\varepsilon_j}) \quad U = \sum_{j=1}^{M} \frac{\varepsilon_j z\mathrm{e}^{-\beta\varepsilon_j}}{1 + sz\mathrm{e}^{-\beta\varepsilon_j}}$$

$$\langle N \rangle = \sum_{j=1}^{M} \frac{sz\mathrm{e}^{-\beta\varepsilon_j}}{1 + sz\mathrm{e}^{-\beta\varepsilon_j}} = \sum_{j=1}^{M} \langle n_j \rangle \quad U = \sum_{j=1}^{M} \langle n_j \rangle \varepsilon_j$$

$$\langle n_i \rangle = \frac{1}{\mathrm{e}^{\beta(\varepsilon_i - \mu)} + s}$$

A. Termodinàmica

Sistema aïllat, $S(U, V, N_j)$:

$$T\mathrm{d}S = \mathrm{d}U + p\mathrm{d}V - \sum_j \mu_j \mathrm{d}N_j$$

Sistema en eq. amb bany tèrmic, $F(T, V, N_j)$

$$F = U - TS \;\rightarrow\; \mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V + \sum_j \mu_j \mathrm{d}N_j$$

Eq. amb bany tèrmic i partícules, $\Xi(T, V, \mu_j)$:

$$\Xi = U - TS - \sum_j \mu_j N_j$$

$$\mathrm{d}\Xi = -S\mathrm{d}T - p\mathrm{d}V - \sum_j N_j \mathrm{d}\mu_j$$

$$\Xi = -pV$$

Energia lliure de Gibbs

$$G(p, T, N) = U - TS + pV = \mu N$$

$$\mathrm{d}G = V\mathrm{d}p - S\mathrm{d}T - \mu\mathrm{d}N$$

B. Probabilitat i estadística

Moments:

$$\langle x^n \rangle = \sum_i x_i^n p(x_i) = \int_{\mathcal{D}} \mathrm{d}x p(x) x^n$$

$$(\Delta x)^2 = \sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

"Funció de partició":

$$Z(\alpha) \equiv \int_0^\infty \mathrm{d}x \mathrm{e}^{-\alpha x} \quad \langle x \rangle = -\frac{\mathrm{d}}{\mathrm{d}\alpha} \log Z(\alpha) \quad \sigma_x^2 = \frac{\mathrm{d}^2}{\mathrm{d}\alpha^2} \log Z(\alpha)$$

Gaussiana:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$\langle x^n \rangle = \int_{-\infty}^{\infty} \mathrm{d}x x^n \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right] = \frac{2^{n/2}}{\sqrt{\pi}} \Gamma\left(\frac{n}{2} + \frac{1}{2}\right)$$

C. Sumes

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad x < 1, \quad \sum_{n=0}^{N} x^n = \frac{x^{N+1} - 1}{x - 1}$$

D. Alguns Sistemes

Magnètics:

$$M = (2n - N)\mu_0 \quad E = -(2n - N)\mu_0 B$$

Polímers:

$$E = -nfl \quad L_c = Nl \quad L = nl$$

Gas ideal:
Clàssic:

$$Z_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \left(\frac{2\pi m}{\beta}\right)^{3N/2}$$

Ultrarrelativista:

$$Z_N = \frac{1}{N!} \left(\frac{8\pi V}{(h\beta c)^3}\right)^N$$

Densitat d'estats:
Cristall bidimensional:

$$\mathrm{d}\Omega = \frac{4\pi m A}{h^2} \mathrm{d}\varepsilon$$

Gas de fotons/neutrins:

$$\mathrm{d}\Omega = \frac{8\pi V}{h^3 c^3} \varepsilon^2 \mathrm{d}\varepsilon$$

Gas d'electrons:

$$\mathrm{d}\Omega = \frac{8\pi m V}{h^3} (2m\varepsilon)^{1/2} \mathrm{d}\varepsilon \qquad N = \int_0^{\varepsilon_F} g(\varepsilon) \mathrm{d}\varepsilon$$

E. Validesa Estadística Clàssica

$$\lambda = \left(\frac{h^2}{2\pi m k_B T}\right)^{1/2} \qquad \lambda/\ell \ll 1$$