Formulario de Física de Medios Continuos

Manel Bosch Aguilera

I. FUNDAMENTOS

$$\vec{F} = \oint_S \vec{\vec{\sigma}} \cdot d\vec{S} \qquad F_i = \int_S \sigma_{ij} n_j dS \qquad \sigma_{ij} = \frac{dF_i}{dS_j}$$

Irrot : $\vec{\nabla} \times \vec{v} = \vec{0} \;\rightarrow\; \vec{v} = \vec{\nabla}\phi$ Incompr: $\vec{\nabla} \cdot \vec{v} = 0 \;\rightarrow\; \vec{v} = \vec{\nabla} \times \vec{A} \;\rightarrow\; \vec{\nabla} \times \vec{v} = \vec{\omega}$

Flujo plano: $v_x = \frac{\partial \psi}{\partial y} \;\; v_y = -\frac{\partial \psi}{\partial x} \;\rightarrow\; \vec{v} \cdot \vec{\nabla}\psi = 0$ Lin. de corriente: $\frac{dx}{dv_x} = \frac{dy}{dv_y} = \frac{dz}{dv_z}$

Ec. trayectoria : $\frac{d\vec{r}}{dt} = \vec{v}(\vec{r}, t)$

II. ELASTICIDAD

Def. lineales: $F = YA\frac{\Delta l}{l} \quad \frac{\Delta w}{w} = \frac{\Delta h}{h} = -\sigma\frac{\Delta l}{l}$ Compresión: $\frac{\Delta V}{V} = -3\frac{(1-2\sigma)}{Y}p$

Cizalla: $\frac{\Delta D}{D} = 2\frac{1+\sigma}{Y}\frac{F}{A} = \frac{1}{\mu}\frac{F}{A} \qquad \frac{F}{A} = \mu\theta \quad \mu = \frac{Y}{2(1+\sigma)}$ Torsión: $\tau = \mu\frac{\pi R^4}{2L}\phi$

Tensor deformaciones

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right) \approx \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$$

$$u_{ij} = \nabla_i v_j = \left[\frac{1}{3}\mathrm{tr}(\vec{\nabla}\vec{v})\vec{\vec{1}}\right] + \left[\frac{1}{2}(\nabla_i v_j + \nabla_j v_i) - \frac{1}{3}\mathrm{tr}(\vec{\nabla}\vec{v})\vec{\vec{1}}\right] + \left[\frac{1}{2}(\nabla_i v_j - \nabla_j v_i)\right]$$

Hooke:

$$\sigma_{ik} = \frac{Y}{1+\sigma}\left(u_{ik} + \frac{\sigma}{1-2\sigma}u_{ll}\delta_{ik}\right) \qquad u_{ik} = \frac{1}{Y}\left[(1+\sigma)\sigma_{ik} - \sigma\sigma_{ll}\delta_{ik}\right]$$

Navier:

$$\frac{Y(1-\sigma)}{(1+\sigma)(1-2\sigma)}\vec{\nabla}(\vec{\nabla} \cdot \vec{u}) - \frac{Y}{2(2+\sigma)}\vec{\nabla} \times (\vec{\nabla} \times \vec{u}) + \vec{f} = \rho\frac{d\vec{v}}{dt}$$

Equilibrio:

$$\vec{f} + \vec{\nabla} \cdot \vec{\vec{\sigma}} = \rho\frac{d\vec{v}}{dt}$$

III. DINÁMICA FLUIDOS

$$M = \int_\Omega \rho(\vec{r}, t) dV \;\rightarrow\; \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho\vec{v}) = 0$$

$$\vec{p} = \int_\Omega \rho\vec{v} dV \;\rightarrow \rho\frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \vec{\nabla})\vec{v} = \vec{\nabla} \cdot \vec{\vec{\sigma}} + \rho\vec{f} \;\rightarrow\; \oint_S [-\rho\vec{v} \cdot \vec{v} + \vec{\vec{\sigma}} + \rho\varphi\vec{\vec{1}}] d\vec{S} = 0$$

$$\vec{\vec{\sigma}} - p\vec{\vec{1}} + \vec{\vec{\Pi}} \qquad \vec{\vec{\Pi}} = 2\eta\left[\frac{1}{2}(\vec{\nabla} \cdot \vec{v} + \vec{\nabla} \cdot \vec{v}^T) - \frac{1}{3}\vec{\nabla} \cdot \vec{v}\vec{\vec{1}}\right] + \eta_v \qquad \eta = \rho\nu$$

Fluido Newtoniano:

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho\vec{v} \cdot \vec{\nabla}\vec{v} = -\vec{\nabla}p + \eta^2\nabla^2\vec{v} + \xi\vec{\nabla} \cdot \vec{\nabla}\vec{v} + \rho\vec{f} \qquad \xi = \eta + \frac{1}{3}\eta_v$$

$$Q = S_1 v_1 = S_2 v_2 \qquad \mathrm{Re} = \frac{uR}{\nu}$$

IV. FLUIDOS IDEALES

$$\eta \nabla^2 \vec{v} = 0 \qquad \rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} p + \rho \vec{f} \qquad \vec{\vec{\sigma}} = -p \vec{\vec{1}} \qquad \vec{f} = -\vec{\nabla} \psi$$

Teorema Boernouilli:

$$\vec{\nabla} \left(\frac{1}{2} \vec{v}^2 + h + \psi \right) = \vec{v} \times \omega \qquad \frac{1}{2} \vec{v}^2 + p + \rho \psi = C \text{ (incompr+irrot)}$$

Teormea Kelvin (ideal, $f_e = 0$):

$$\Gamma = \oint_C \vec{v} \cdot \mathrm{d}\vec{l} \rightarrow \frac{\mathrm{d}\Gamma}{\mathrm{d}t} = 0$$

Stokes:

$$\oint_{C(t)} \vec{v} \cdot \mathrm{d}\vec{l} = \int_{S_c(t)} (\vec{\nabla} \times \vec{v}) \cdot \mathrm{d}\vec{S} = \int_{S_c(t)} \vec{\omega} \cdot \mathrm{d}\vec{S}$$

FluJos potencialEs (2D):

Fuente / sumidero: $\vec{v} = \frac{q}{2\pi r} \hat{e}_r \qquad \phi = \frac{q}{2\pi} \ln r \qquad \psi = \frac{q}{2\pi} \theta$

Uniforme: $\vec{v} = v \hat{e}_x \qquad \phi = v_x \qquad \psi = v_y$

Circulación:

$$v_r = 0 \qquad \vec{v}_\theta = g(r) \hat{e}_\theta \quad \Gamma = 2\pi r g(r) \qquad \vec{v}_\theta = \frac{\Gamma}{2\pi r} \hat{e}_\theta \qquad \phi = \frac{\Gamma \phi}{2\pi}$$

$$\psi = -\frac{\Gamma}{2\pi} \ln r \qquad \frac{1}{r} \hat{e}_z \left[\frac{\partial}{\partial r} (r v_\theta) \right] = 0$$

Dipolo: $\phi = -\frac{\vec{p} \cdot \hat{r}}{2\pi r}$

Flujos potenciales (3D):

Uniforme: $\phi = ur \cos\theta \qquad \psi = -\frac{ur^2}{2} \sin^2 \theta$

Fuente/sumidero:

$$v_r = \frac{Q}{4\pi r^2} \quad \rightarrow \quad \phi = -\frac{Q}{4\pi r} \qquad \psi = -\frac{Q}{4\pi} \cos\theta$$

Dipolo:

$$\phi = -\frac{\vec{p} \cdot \hat{r}}{4\pi r^2} \quad v_r = \frac{p \cos\varphi}{2\pi r^3} \quad v_\varphi = \frac{p_0 \sin\varphi}{4\pi r^2} \qquad \psi = \frac{p_0 \sin\varphi}{4\pi r}$$

V. FLUIDOS VISCOSOS

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\vec{\nabla} p + \eta \nabla^2 \vec{v} + \rho \vec{f}$$