$$\frac{h\nu'}{h\nu} = \frac{mc^2}{mc^2+mgh} \approx 1 - gh/c^2$$

$$\Delta T_h \approx (1+gh/c^2)\Delta T_0$$

$$\frac{1}{L^2} = \frac{GM}{r^3c^2}$$

$$\vec{v} = \frac{\mathrm{d}}{\mathrm{d}\lambda} = \sum_i \left.\frac{\mathrm{d}x^i}{\mathrm{d}\lambda}\right|_P \frac{\partial}{\partial x^i} = \sum_i v^i \frac{\partial}{\partial x^i}$$

$$[\vec{X},\vec{Y}] = \sum_{i,j}^{n}\left(x^i\frac{\partial y^j}{\partial x^i} - y^j\right)\frac{\partial}{\partial x^j}$$

$$x^i(\lambda_0+\epsilon) = \left.\mathrm{e}^{\epsilon\frac{\mathrm{d}}{\mathrm{d}\lambda}}\right|_{\lambda_0} x^i = \left.\mathrm{e}^{\epsilon\vec{X}}\right|_{\lambda_0} x^i$$

$$\tilde{\nabla} f\left(\frac{\mathrm{d}}{\mathrm{d}\lambda}\right) := \left.\frac{\mathrm{d}f}{\mathrm{d}\lambda}\right|_p \quad , \quad \tilde{\nabla} f = \frac{\partial f}{\partial x^i}\tilde{\nabla} x^i$$

$$\vec{e}_{i'} = \Lambda^i_{\ i'}\vec{e}_i\ , \quad \tilde{\omega}^{i'} = \Lambda^{i'}_{\ i}\tilde{\omega}^i\ , \quad v^{i'} = \Lambda^{i'}_{\ i}v^i\ , \quad p_{i'} = \Lambda^i_{\ i'}p_i$$

$$g' = \Lambda^T g \Lambda$$

Subvariedad $\forall p \in S, \exists(\phi, U(p))$ tq $\phi : U \cap \mathbb{R}^{(n,m)} \to V \cap S$

$\phi : \mathbb{M}(x_i) \to \mathbb{N}(y_i$

Push-fw $\phi^* : T_x \to T_{\phi(x)}$, $\vec{v}^*(f) = \vec{v}(f\circ\phi)$, $v^{*\alpha} = v^i\frac{\partial y^\alpha}{\partial x^i}$

Pull-back $\phi_* : T_{\phi(x)}x \to T_x$, $\omega^*(\vec{v}) = \omega(\vec{v}^*)$, $\omega^*_i = \omega_\alpha\frac{\partial y^\alpha}{\partial x^i}$

Transporte Lie $g_{\Delta t}(x^i(t)) = x^i(t+\Delta t)$

Derivada Lie

$$\pounds_{\vec{v}}\, f := \lim_{\Delta t\to 0}\frac{f^*_{\Delta t}(t_0) - f(t_0)}{\Delta t} = \vec{v}(f)$$

$$\pounds_{\vec{u}}\, \vec{v} := \lim_{\Delta t}\frac{(g^{-1}_{\Delta t})^*\vec{v}(g_{\Delta t}\vec{x}) - \vec{v}(\vec{x})}{\Delta t} = [\vec{u},\vec{v}]$$

$$\pounds_{\vec{u}}\,(\tilde{\omega}) := \lim_{\Delta t\to 0}\frac{(g^*_{\Delta t})\tilde{\omega}(g_{\Delta t}\vec{x}) - \tilde{\omega}(x^i)}{\Delta t} = (u^k\partial_k\omega_j + \omega_k\partial_j u^k)\tilde{\nabla} x^j$$

$$\pounds_{\vec{u}}\,\big(\tilde{\omega}(\vec{v})\big) = \big(\pounds_{\vec{u}}\,(\tilde{\omega})\big)\vec{v} + \tilde{\omega}\big(\pounds_{\vec{u}}\,(\vec{v})\big) \quad , \quad \pounds_{[\vec{x},\vec{y}]} = \big[\pounds_{\vec{x}}\,, \pounds_{\vec{y}}\,\big]$$

$$\begin{aligned}(\pounds_{\vec{u}}\,T)^{i_1,\dots,i_p}_{j_1,\dots,j_q} &= u^k\frac{\partial}{\partial x^k}T^{i_1,\dots,i_p}_{j_1,\dots,j_q} \\ &+ T^{i_1,\dots,i_p}_{k,j_2,\dots,j_q}\frac{\partial u^k}{\partial x^{j_1}} + \dots + T^{i_1,\dots,i_p}_{j_1,\dots,j_{q-1},k}\frac{\partial i^k}{\partial x^{j_q}} \\ &- T^{k,i_2,\dots,i_p}_{j_1,\dots,j_q}\frac{\partial u^{u_1}}{\partial x^k} - T^{i_1,\dots,i_{p-1},k}_{j_1,\dots,j_q}\frac{\partial u^{i_p}}{\partial x^k}\end{aligned}$$

Killing $\pounds_{\vec{x}}\, g = 0$, max $n(n+1)/2$

$$\nabla_{(\alpha}\xi_{\beta)} = 0$$

$$\partial_j x^i + \partial_i x^j = 2x_l\Gamma^l_{\ ji}$$

$$\tilde{p}\wedge\tilde{q} := \tilde{p}\otimes\tilde{q} - \tilde{q}\otimes\tilde{p} \quad , \quad \tilde{\alpha} = \frac{1}{p!}\alpha_{[i\dots j]}\ \tilde{\omega}^i\wedge\dots\wedge\tilde{\omega}^j$$

$$\tilde{p}\wedge\tilde{q} = \frac{1}{p!q!}p_{i\dots j}\ q_{k\dots l}\tilde{\omega}^i\wedge\dots\wedge\tilde{\omega}^l = \frac{1}{(p+q)!}(\tilde{p}\wedge\tilde{q})_{i\dots l}\tilde{\omega}^i\wedge\dots\wedge\tilde{\omega}^l$$

$$\tilde{p}\wedge\tilde{q} = (-1)^{pq}\tilde{q}\wedge\tilde{p}$$

$$(i_{\vec{x}})\tilde{\omega} = \tilde{\omega}(\vec{x},\ ,\dots,\ ) = \frac{1}{(p-1)!}x^i p_{ij\dots k}\tilde{\omega}^j\wedge\dots\wedge\tilde{\omega}^k$$

$$\det A = \epsilon_{il\dots j}A^{1i}A^{2l}\cdots A^{nj} = \frac{1}{n!}\epsilon_{ab\dots c}\epsilon_{il\dots j}A^{ai}A^{bl}\cdots A^{cj}$$

$$\tilde{\omega} = \frac{1}{n!}f\epsilon_{ij\dots k}\tilde{\omega}^i\wedge\tilde{\omega}^j\wedge\dots\wedge\tilde{\omega}^k \quad , \quad f' = f\det\Lambda$$

$$\tilde{\omega} = \tilde{\omega}^1\wedge\dots\wedge\tilde{\omega}^n = \sqrt{|g|}\tilde{\nabla}x^1\wedge\dots\wedge\tilde{\nabla}x^n$$

$$\text{dual: } (*\alpha)_{i\dots j} = \frac{1}{q!}\omega_{k\dots li\dots j}\alpha^{k\dots l}$$

$$*(\tilde{\omega}^r\wedge\dots\wedge\tilde{\omega}^t) = \frac{1}{(n-p)!}\ g^{ri}\cdots g^{tk}\ \omega_{i\dots kl\dots m}\tilde{\omega}^l\wedge\dots\wedge\tilde{\omega}^m$$

$$\tilde{\mathrm{d}}(\tilde{\alpha}+\tilde{\beta}) = \tilde{\mathrm{d}}\tilde{\alpha}+\tilde{\mathrm{d}}\tilde{\beta} \quad , \quad \tilde{\mathrm{d}}(\tilde{\alpha}\wedge\tilde{\beta}) = \tilde{\mathrm{d}}\tilde{\alpha}\wedge\tilde{\beta}+(-1)^\alpha\tilde{\alpha}\wedge\tilde{\mathrm{d}}\tilde{\beta}$$

$$\tilde{\mathrm{d}}\tilde{\mathrm{d}}\tilde{\alpha} = 0 \quad , \quad \tilde{\mathrm{d}}(f) = \tilde{\nabla} f$$

$$\tilde{\mathrm{d}}\tilde{\alpha} = \frac{1}{\alpha!}\frac{\partial\alpha_{i\dots j}}{\partial x^m}\ \tilde{\nabla}x^m\wedge\tilde{\nabla}x^i\wedge\dots\wedge\tilde{\nabla}x^j$$

$$\mathrm{rot}\tilde{p} = *\tilde{\mathrm{d}}\tilde{p} \quad , \quad \mathrm{div}\tilde{p} = \tilde{\mathrm{d}}*\tilde{p}$$

$$\int_\Omega\tilde{F} = \int_\Omega\tilde{F}(\mathrm{e}_1,\dots,\mathrm{e}_n)\tilde{\nabla}x^1\wedge\dots\wedge\ \tilde{\nabla}x^k :=$$

$$:= \int_U\tilde{F}(\mathrm{e}_1,\dots,\mathrm{e}_n)\mathrm{d}x^1\cdots\mathrm{d}x^k$$

$$\text{Stokes } \int_\Omega\tilde{\mathrm{d}}\tilde{F} = \int_{\partial\Omega}\tilde{F}$$

$$\text{Gauss } \int_\Omega\mathrm{d}(\tilde{\omega}(\vec{A})) = \int_{\partial\Omega}\tilde{\omega}(\vec{A})$$

$$\int_\Omega\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\alpha}(A^\alpha\sqrt{g})\tilde{\omega} = \int_{\partial\Omega}A^\alpha n_\alpha\tilde{s}$$

$$\nabla_{\vec{u}}\vec{v} = \lim_{\epsilon\to 0}\frac{\vec{v}^*_{\lambda_0+\epsilon}(\lambda_0) - \vec{v}(\lambda_0)}{\epsilon} \quad , \quad \nabla_{\mathrm{e}_j}\mathrm{e}_i = \Gamma^k_{ij}\mathrm{e}_k$$

1. $\nabla_{\vec{u}}(f\vec{v}) = (\nabla_{\vec{u}}f)\vec{v} + f(\nabla_{\vec{u}}\vec{v})$
2. $\nabla_{\vec{u}}(f) = \vec{u}(f)$
3. $\nabla_{\vec{u}}(\vec{A}\cdot\vec{B}) = (\nabla_{\vec{u}}\vec{A})\cdot\vec{B} + \vec{A}\cdot(\nabla_{\vec{u}}\vec{B})$
4. $\nabla_{\vec{u}}(\tilde{\sigma}(\vec{A})) = (\nabla_{\vec{u}}\tilde{\sigma})(\vec{A}) + \tilde{\sigma}(\nabla_{\vec{u}}\vec{A})$
5. $\nabla_{f\vec{u}+g\vec{v}}\vec{A} = f\nabla_{\vec{u}}\vec{A} + g\nabla_{\vec{v}}\vec{A}$

$$\nabla_{\vec{u}}\vec{v} = u^i\Big(\mathrm{e}_i(v^k) + v^j\Gamma^k_{ji}\Big)\mathrm{e}_k = u^i(\nabla_i v^k)\mathrm{e}_k = u^i v^k_{;i}\mathrm{e}_k$$

$$\nabla_{\mathrm{e}_k}\tilde{\omega}^j = -\Gamma^j_{lk}\tilde{\omega}^l \quad , \quad \nabla_i\sigma_j = \partial_i\sigma_j - \Gamma^k_{ij}\sigma_k$$

$$\begin{aligned}\nabla_m T^{i\ldots j}_{k\ldots l} = {} & \partial_m T^{i\ldots j}_{k\ldots l} + T^{a\ldots j}_{k\ldots l}\Gamma^i_{am} + \cdots + T^{i\ldots a}_{k\ldots l}\Gamma^j_{am} \\ & - T^{i\ldots j}_{a\ldots l}\Gamma^a_{km} - \ldots - T^{i\ldots j}_{k\ldots a}\Gamma^a_{lm}\end{aligned}$$

$$T^k_{ij} := \nabla_{\mathrm{e}_j}\mathrm{e}_i - \nabla_{\mathrm{e}_i}\mathrm{e}_j - [\mathrm{e}_i,\mathrm{e}_j] = 0 \Leftrightarrow \Gamma^k_{ij} = \Gamma^k_{ji}$$

$$(\pounds_{\vec{u}}\,\vec{v})^i = u^j\partial_j v^i - v^j\partial_j u^i = u^j\nabla_j v^i - v^j\nabla_j u^i$$

$$(\pounds_{\vec{u}}\,\tilde{\sigma})_i = u^j\partial_j\sigma_i + \sigma_k\partial_i u^k = u^j\nabla_j\sigma_i + \sigma_k\nabla_i u^k$$

Geodésicas $\nabla_{\vec{u}}(\vec{u}) = 0 \quad , \quad \nabla_{\vec{u}}(\vec{u}) = f\vec{u}$

$$\frac{\partial^2 x^k}{\partial\lambda^2} + \Gamma^k_{ji}\frac{\partial x^i}{\partial\lambda}\frac{\partial x^j}{\partial\lambda} = 0$$

trans. paral. $\nabla_{\vec{u}}\vec{v} = 0 \to u^i\left(\frac{\partial v^k}{\partial x^i} + v^j\Gamma^k_{ij}\right) = 0$

Curvatura $\mathbb{R}(\vec{u},\vec{v}) = [\nabla_{\vec{u}},\nabla_{\vec{v}}] - \nabla_{[\vec{u},\vec{v}]}$

$$R^m_{kij} = \partial_i\Gamma^m_{kj} - \partial_j\Gamma^m_{ki} + \Gamma^l_{kj}\Gamma^m_{li} - \Gamma^l_{ki}\Gamma^m_{lj} - C^l_{ij}\Gamma^m_{kl}$$

$$\Gamma^i_{jk} = \frac{1}{2}g^{il}(g_{lj,k} + g_{lk,j} - g_{jk,l})$$

- $\nabla_i\nabla_j V^l - \nabla_j\nabla_i V^l = R^l_{kij}V^k$
- $R^m_{kij} = -R^m_{kji} \Leftrightarrow R^m_{k(ij)} = 0$
- $R^m_{[kij]} = 0$
- $\# = n^2(n^2-1)/3$
- Bianchi $\nabla_{[m}R^l_{|k|ij]} = R^l_{k[ij;m]} = 0$
- $R_{ijkl} = -R_{ijlk}$
- $R_{ijkl} = R_{klij}$
- $R_{[ijkl]} = 0$
- $\# = n^2(n^2-1)/12$

$$[u,v] = 0 \Leftrightarrow u^j u^k\nabla_j\nabla_k v^l = R^i_{jkl}u^j u^k v^l$$

$$\nabla_i v^i = \mathrm{d}\tilde{\omega}(\vec{v}) \Rightarrow \Gamma^i_{ji} = \partial_j\ln\sqrt{g}$$

$$S = \int g_{ij}\dot{x}^i\dot{x}^j\mathrm{d}\lambda \quad , \quad \delta S = 0 \Leftrightarrow \ddot{x}^k + \Gamma^k_{ij}\dot{x}^i\dot{x}^j = 0$$

$T^{\alpha\beta} =$ flux p^α per $x^\beta =$ cte

Polvo $n = N/\Delta V$, $n' = \gamma n$

$$\vec{N} = n\vec{u} = (\gamma nc, \gamma nv) \quad , \quad T^{00} = \rho = nm$$

$$\mathbb{T} = \vec{N}\otimes\vec{p} \quad , \quad T^{\mu\nu} = nmu^\mu u^\nu$$

Fluido ideal

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$$

$$a^i = u^\beta\nabla_\beta u^i = -\frac{\partial^i p}{\rho + p}$$

n partículas

$$T^{\alpha\beta} = \sum_N\int \mathrm{d}\tau p^\alpha_N\frac{\mathrm{d}x^i_N}{\mathrm{d}\tau}\delta^4\big(\vec{x} - \vec{x}_N(t)\big)$$

$$p = \frac{1}{3}\sum\frac{\vec{p}^2_N}{E_N}\delta^3(\vec{x} - \vec{x}_N(t))$$

$$\rho = \frac{1}{3}\sum E_N\delta^3(\vec{x} - \vec{x}_N(t))$$

(no rel) $\rho = nm + \frac{3}{2}p$, (molt rel) $\rho = 3p$

$$G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = \kappa T^{\alpha\beta} \quad , \quad \kappa = \frac{8\pi G}{c^4}$$

Límite Newton $T_{i\alpha} \to 0$, $\partial_0 \to 0$

$$h_{00} = \frac{2\phi}{c^2} \quad , \quad R_{00} = -\frac{1}{2}\nabla^2 h_{00}$$

Gauge $x^{\alpha'} = x^\alpha + t\xi^\alpha(x)$, $t \ll 1$

$$\hat{h}_{\alpha\beta} = h_{\alpha\beta} - \left(\partial_\beta \xi_\alpha + \partial_\alpha \xi_\beta\right) = h_{\alpha\beta} - \pounds_{\vec{\xi}}(\eta_{\alpha\beta})$$

$$R_{\alpha\beta\mu\nu} = \frac{1}{2}\Big(\partial_\beta\partial_\mu h_{\alpha\nu} + \partial_\alpha\partial_\nu h_{\beta\mu} - \partial_\beta\partial_\nu h_{\alpha\mu} - \partial_\alpha\partial_\mu h_{\beta\nu}\Big)$$

$$R_{\mu\nu} = \frac{1}{2}\Big(\partial_\mu\partial_\alpha h^\alpha_\nu + \partial_\alpha\partial_\nu h^\alpha_\mu - \partial_\mu\partial_\nu h^\alpha_\alpha - \partial_\alpha\partial^\alpha h_{\mu\nu}\Big)$$

$$R = \partial_\beta\partial_\alpha h^{\alpha\beta} - \partial_\beta\partial^\beta h^\alpha_\alpha$$

pot $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\alpha_\alpha$

g.Lorentz $\partial^\mu \bar{h}_{\mu\nu} = 0$, $\Box^2 \xi_\nu = \partial^\mu h_{\mu\nu}$

$$\hat{\bar{h}}_{\mu\nu} = \bar{h}_{\mu\nu} - \left(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu\right) + \eta_{\mu\nu}\partial_\alpha\xi^\alpha$$

$$\Box^2 \bar{h}_{\mu\nu} = -2\kappa T_{\mu\nu}$$

$l.N$ $\bar{h}_{00} = -4\phi$, $\mathrm{d}s^2 = -(1+2\phi)\mathrm{d}t^2 + (1-2\phi)(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$

Landau-Lifshitz $\partial_\mu\Big[(-g)\big(T^{\mu\nu} + t^{\mu\nu}\big)\Big] = 0$

$$t^{\mu\nu} = \frac{1}{2\kappa(-g)}\partial_\alpha\partial_\beta\left[(-g)\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}\right)\right] - \frac{1}{\kappa}G^{\mu\nu}$$

$$(-g)\big(G^{\mu\nu} + \kappa t^{\mu\nu}\big) = \partial_\alpha u^{\mu\nu\alpha}$$

$$\partial_\beta u^{\mu\nu\alpha\beta} = u^{\mu\nu\alpha} \longrightarrow (-g)\big(G^{\mu\nu} + \kappa t^{\mu\nu}\big) = \partial_\alpha\partial_\beta u^{\mu\nu\alpha\beta}$$

$$P^\mu := \int_\Sigma \mathrm{d}^3x\, \frac{\partial_\alpha u^{\mu 0\alpha}}{\kappa} = \int_{\partial\Sigma}\mathrm{d}^2S_i\, \partial_\beta \frac{(-g)\left(g^{\mu 0}g^{i\beta} - g^{\mu i}g^{0\beta}\right)}{2\kappa}$$

Gauge TT $h_{\alpha 0} = 0$, $h^\alpha_\alpha = 0$, $\partial^i h_{ik} = 0$

$$\bar{h}_{\mu\nu} = \mathrm{Re}\left(A_{\mu\nu}\, \mathrm{e}^{ik_\alpha x^\alpha}\right) \quad , \quad k_\alpha k^\alpha = k^\alpha A_{\mu\alpha} = A^\mu_\mu = 0$$

$$R_{j0k0} = R_{0j0k} = -\frac{1}{2}\partial_0\partial_0 h^{TT}_{jk}$$

$$x^{\hat{i}}_A = 0 \quad , \quad x^{\hat{i}}_B = x^j_B\left(\delta^{\hat{i}}_j + \frac{1}{2}h^{\hat{i}}_j(t, x^i = 0)\right)$$

$$e_+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad e_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad k^\alpha = \omega(1,0,0,1)$$

$$h^{TT}_{ij} = \mathrm{Re}\left\{A_+ e_+ \mathrm{e}^{-i\omega(t-z)} + A_X e_X \mathrm{e}^{-i\omega(t-z)}\right\}$$

$$h^{TT}_{ij} = P_{il}h_{lm}P_{mj} - \frac{1}{2}P_{ij}P_{ij}(P_{lm}h_{lm}) \quad , \quad P_{lm} = \delta_{lm} - k_l k_m$$

$$\left\langle h\ h_{\mu\nu|\alpha\beta}\right\rangle = \left\langle h\ h_{\alpha\beta|\mu\nu}\right\rangle \ , \ \left\langle \left(h_{|\alpha}\ h_{\mu\nu}\right)_{|\beta}\right\rangle = 0$$

$$\left\langle h\ h_{\mu\nu|\alpha\beta}\right\rangle = -\left\langle h_{|\beta}\ h_{\mu\nu|\alpha}\right\rangle$$

$$T^{OG}_{\mu\nu} = \frac{1}{4\kappa}\left\langle h_{\alpha\beta|\mu}h^{\alpha\beta}{}_{|\nu}\right\rangle \quad , \quad T^{\mu OG}_\mu = 0$$

$$R_{\mu\nu} = R^{(F)}_{\mu\nu} + \underbrace{R^{(1)}_{\mu\nu}(h^{(1)})}_{o(A/\lambda^2)} + \underbrace{R^{(1)}_{\mu\nu}(h^{(2)}) + R^{(1)}_{\mu\nu}(h^{(2)})}_{o(A^2/\lambda^2)} + o(A^3/\lambda^2)$$

$$R^{(1)}_{\mu\nu}(h) = \frac{1}{2}\left(h_{\alpha\mu|\nu}{}^\alpha + h_{\alpha\nu|\mu}{}^\alpha - h^\alpha_{\alpha|\mu\nu} - h_{\mu\nu|\alpha}{}^\alpha\right)$$

$$\begin{aligned} R^{(2)}_{\mu\nu}(h) = \frac{1}{2}\bigg\{ & \frac{1}{2}h_{\alpha\beta|\mu}h^{\alpha\beta}{}_{|\nu} + h^{\alpha\beta}\left(h_{\alpha\beta|\mu\nu} + h_{\mu\nu|\alpha\beta}\right. \\ & \left. - h_{\alpha\mu|\nu\beta} - h_{\alpha\nu|\mu\beta}\right) + h_\nu{}^{\alpha|\beta}\left(h_{\alpha\mu|\beta} - h_{\beta\mu|\alpha}\right) \\ & - \left(h^{\alpha\beta}{}_{|\beta} - \frac{1}{2}h^{|\alpha}\right)\left(h_{\alpha\mu|\nu} + h_{\alpha\nu|\mu} - h_{\mu\nu|\alpha}\right)\bigg\} \end{aligned}$$

$$\tilde{\bar{h}}_{ij} = -\frac{k}{4\pi}\omega^2\frac{\mathrm{e}^{i\omega r}}{r}\tilde{D}_{ij} \quad \text{on} \quad \tilde{D}_{ij} = \int \tilde{T}^{00}x^{i'}x^{j'}\mathrm{d}^3x'$$

$$\bar{h}_{ij} = \frac{k}{4\pi r}\frac{\mathrm{d}^2}{\mathrm{d}t^2}D_{ij}(t-r) \quad \text{on} \quad D_{ij} = \int T^{00}x^{i'}x^{j'}\mathrm{d}^3x'$$

$$\bar{h}^{TT}_{ij} = \frac{c^4}{4\pi r}\frac{\mathrm{d}^2}{\mathrm{d}t^2}I^{TT}_{ij} = \frac{2G}{r}\frac{\mathrm{d}^2}{\mathrm{d}t^2}I^{TT}_{ij}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = r^2 \cdot n_i T^{0i} = -\frac{n^i}{4\pi}r^2\left\langle h^{TT}_{jk,0}h^{TT\ jk}_{,i}\right\rangle$$

$$I_{ij} = D_{ij} - \frac{1}{3}\delta_{ij}D^k_k = \int \rho\left(x^{i'}x^{j'} - \frac{1}{3}\delta^{ij}r'^2\right)\mathrm{d}^3x'$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\kappa}{64\pi^2 c}\left\langle \dddot{I}_{TTjk}\dddot{I}_{TTjk}\right\rangle = \frac{G}{8\pi c^5}\left\langle \dddot{I}_{TTjk}\dddot{I}_{TTjk}\right\rangle$$

$$P = \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\kappa}{40\pi c}\left\langle \dddot{I}_{TTjk}\dddot{I}_{TTjk}\right\rangle = \frac{G}{5c^5}\left\langle \dddot{I}_{TTjk}\dddot{I}_{TTjk}\right\rangle$$

$$\mathrm{d}s^2 = -\mathrm{e}^{2\phi(t,r)}\mathrm{d}t^2 + \mathrm{e}^{2\Lambda(t,r)}\mathrm{d}r^2 + (\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

Schwarchild $\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r^2\mathrm{d}^2\Omega$

$$\begin{array}{lll}
\Gamma^1_{11} = \Lambda' & , \ \Gamma^1_{33} = -r\sin^2\theta\mathrm{e}^{-2\Lambda} & , \ \Gamma^3_{23} = \mathrm{cotg}\theta \\
\Gamma^1_{10} = \dot\Lambda & , \ \Gamma^2_{12} = 1/r & , \ \Gamma^0_{11} = \dot\Lambda\mathrm{e}^{2(\Lambda-\phi)} \\
\Gamma^1_{22} = -r\mathrm{e}^{-2\Lambda} & , \ \Gamma^2_{33} = -\sin\theta\cos\theta & , \ \Gamma^0_{10} = \phi' \\
\Gamma^1_{00} = \mathrm{e}^{2(\phi-\Lambda)}\phi' & , \ \Gamma^3_{13} = 1/r & , \ \Gamma^0_{00} = \dot\phi
\end{array}$$

$$\begin{aligned}
R_{00} &= \mathrm{e}^{2(\phi-\Lambda)}\left(\phi'' + \phi'^2 - \phi'\Lambda' + \frac{2\phi'}{r}\right) - \ddot\Lambda - \dot\Lambda^2 + \dot\Lambda\dot\phi \\
R_{11} &= -\phi'' - \phi'^2 + \phi'\Lambda' + \frac{2\Lambda'}{r} + \mathrm{e}^{2(\Lambda-\phi)}\left(\ddot\Lambda + \dot\Lambda^2 - \dot\Lambda\dot\phi\right) \\
R_{22} &= -\mathrm{e}^{-2\Lambda}\left[1 + r(\phi'-\Lambda')\right] + 1 \\
R_{33} &= -\sin^2\theta\left\{\mathrm{e}^{-2\Lambda}\left[1 + r(\phi'-\Lambda')\right] - 1\right\} \\
R_{10} &= \frac{2\dot\Lambda}{r}
\end{aligned}$$

$$\mathrm{e}^{2\Lambda} = \frac{1}{1-2\frac{m(r)}{r}} \quad , \quad m(r) = \frac{\kappa}{2}\int_0^r \mathrm{d}r\, r^2\rho(r)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{m(r) + \frac{\kappa}{2}pr^3}{r(r-2m(r))}$$

Oppenheimer-Volkov $\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+\rho)\left(m(r) + 4\pi r^3\rho\right)}{r(r-2m(r))}$

Clàssica $\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho\frac{m(r)}{r}$

$$M_{\mathrm{mat}} = \int_0^{r_0}\sqrt{-g}\big|_{t=\mathrm{cte}}\rho\mathrm{d}^3x = 4\pi\int_0^{r_0}\mathrm{e}^{\Lambda}\rho r^2\sin\theta\mathrm{d}r$$

$$p(r_0) = 0 \quad , \quad r_0 > \frac{9}{4}M \Rightarrow p(o) = \infty$$

$m \neq 0 \rightarrow p_0 = -\hat{E}m$, $p_\varphi = \hat{L}m$

$$\hat{E} = \left(1-\frac{2M}{r}\right)\frac{\mathrm{d}t}{\mathrm{d}\tau}$$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 = \hat{E}^2 - \left(1-\frac{2M}{r}\right)\left(1+\frac{\hat{L}^2}{r^2}\right) = \hat{E}^2 - \hat{V}^2(r)$$

$$r_0 = \frac{\hat{L}^2}{2M}\left(1 \pm \sqrt{1-\frac{12M^2}{\hat{L}^2}}\right)$$

$m = 0 \rightarrow p_0 = -E$, $p_\varphi = L$, $r_0 = 3M$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 = E^2 - \left(1-\frac{2M}{r}\right)\frac{L^2}{r^2} = E^2 - V^2(r)$$

Precesión perihelio $u = 1/r = y + M/\hat{L}^2$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\varphi}\right)^2 = \frac{\hat{E}^2 - (1-2M/r)\left(1+\hat{L}^2/r^2\right)}{\hat{L}^2/r^4}$$

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\varphi}\right)^2 = \frac{\hat{E}^2}{\hat{L}^2} - (1-2Mu)\left(\frac{1}{\hat{L}^2} + u^2\right)$$

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\varphi}\right)^2 = \frac{\hat{E}^2 + (M/\hat{L})^2 + 2(M/\hat{L})^4 - 1}{\hat{L}^2} + \frac{6M^3}{\hat{L}^2}y + \left(\frac{6M^2}{\hat{L}^2} - 1\right)y^2 + (2My^3)$$

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\varphi}\right)^2 = A + By - k^2y^2 \ , \quad k^2 = 1 - \frac{6M^2}{\hat{L}^2}$$

$$y = C\cos(k\varphi + \varphi_0) \quad ,$$

$$\Delta\varphi = \phi - 2\pi = 2\pi\left(\frac{1}{1-\frac{6M^2}{\hat{L}^2}} - 1\right) \approx \frac{6\pi M^2}{\hat{L}^2} = \frac{6\pi M}{r}$$

Deflexión lluz

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\varphi}\right)^2 + (1-2Mu)\,u^2 = \frac{E^2}{L^2} = \frac{1}{b^2}$$

$$M = 0 \Longleftrightarrow u_0(\varphi) = \frac{1}{b}\sin\varphi$$

$$\frac{\mathrm{d}^2u}{\mathrm{d}\varphi^2} + u = 3Mu^2 \approx \frac{3M}{b^2}\sin^2\varphi$$

$$u(\varphi) = \frac{1}{b}\sin\varphi + \frac{M}{b^2}(1+\cos\varphi)^2$$

$$u(\varphi) = \frac{1}{b}\Delta\varphi + \frac{4M}{b^2} = 0 \longrightarrow \Delta\varphi = -\frac{4M}{b}$$